Distribution	PDF/PMF	SUPPORT	EX	VARX	MGF	Notes
Bernoulli(p)	$p^x(1-p)^{1-x}$	X=0,1; 0≤p≤1	p	p(1-p)	$(1-p)\ +\ pe^t$	
Binomial(n,p)	$\binom{n}{x}p^x(1-p)^{n-x}$	X=0,1,…n 0≤p≤1	p	np(1-p)	$[pe^t+(1-p)]^n$	Multinomial Distr. Is multivariate binomial
Discrete Uniform	$\frac{1}{N}$	X= 1,..N; N=1,2,…	(N+1)/2	$(N^2-1)/12$	$\frac{1}{N}\sum e^{it}$	
Geometric (p)	$p(1-p)^{x-1}$	X=1,2,..; 0≤p≤1	1/p	$(1-p)/p^2$	$\frac{pe^t}{1-(1-p)e^t}$	In MGF: t<-log(1-p) Y=X-1 is negative binomial(1,p) Memoryless: $P(X>s \mid X>t)=P(X>s-t)$
Hypergeometric	$\frac{\binom{M}{x}\binom{N-M}{K-k}}{\binom{N}{K}}$	X=0…K; N,M,K≥0; M-(N-K)≤x≤M	(KM)/N	$\frac{KM(N-M)(N-K)}{N^2(N-1)}$		
Negative Binomial (r,p)	$\binom{r+x-1}{x}p^r(1-p)^x$	X=0,1..; 0≤p≤1	r(1-p)/p	$r(1-p)/p^2$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$	Gamma mixture of Poisson's X is trial at which r^{th} success occurs
Possion (λ)	$\frac{e^{-\lambda}\lambda^x}{x!}$	X=0,1,…; 0≤λ≤inf	λ	λ	$e^{\lambda(e^t-1)}$	MGF uses $\sum_0^\infty \frac{(e^t\lambda)^\wedge x}{x!}=e^{e^t\lambda}$ Good approximation to binomial, n>20, p<0.5
Beta (α,β)	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	0≤X≤1; α,β>0	α/(α+β)	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$		Beta(1,1) is Uniform(0,1) B>α is skewed right, B=α is symmetric X~Gam(a,c), Y~Gam(b,c) then X/(X+Y)~Beta(a,b)
Chi-Squared (p)	$\frac{1}{\Gamma\left(\frac{p}{2}\right)2^{\frac{p}{2}}}x^{\frac{p}{2}-1}e^{-\frac{x}{2}}$	0≤X≤inf; p=1,2,..	p	2p	$\left(\frac{1}{1-2t}\right)^{\frac{p}{2}}$	Special case of the gamma (p/2,2), $(n-1)s^2/\sigma^2 \sim \chi_{(n-1)}$ $\sum Z_i^2 \sim \chi_{(n)}$
Exponential (β)	$\left(\frac{1}{\beta}\right)e^{-\frac{x}{\beta}}$	0≤X≤inf; β>0	β	β^2	$\frac{1}{1-\beta t}$; t<1/ β	Special case of the gamma (1, β), Memoryless X~Exp(1) means $\sum X_i \sim$Gamma(n,1)
F	$F_{v_1,v_2}=\frac{\chi^2_{v_1}/v_1}{\chi^2_{v_2}/v_2}$	0≤X≤inf; v_1,v_2=1,2,…	$v_2/(v_2-2)$		Does not exist	Special case of independent chisquares $F_{1,v}=(t_v)^2$ $1/F_{m,n}=F_{n,m}$
Gamma (α,β)	$\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-\frac{x}{\beta}}$	0≤X≤inf; α,β>0	αβ	$\alpha\beta^2$	$\left(\frac{1}{1-\beta t}\right)^\alpha$	Exponential (α=1) and Chi-squared (α=p/2,β=2) $\sum X_i \sim$Gamma($\sum\alpha_i$,β) where $X_i \sim$Gamma(α_i,β) cX~Gamma(α,cβ)
Normal (μ,σ^2)	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\left(\frac{1}{2\sigma^2}\right)(x-\mu)^2}$	-inf≤X,μ≤inf; σ>0	μ	σ^2	$e^{\mu t+\frac{\sigma^2t^2}{2}}$	MGF: complete the square, MGF proves Z~N(0,1), Sums of normals are normal, Z^2~Chi-square$_1$ratio of 2 norms= Cauchy
t	$\frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})}\frac{1}{\sqrt{v\pi}}\frac{1}{\left(1+\left(\frac{x^2}{v}\right)\right)^{((v+1)/2)}}$	-inf≤X≤inf; v>0	0	v/(v-2); v>2		$F_{1,v}=(t_v)^2$ $T=\frac{U}{\sqrt{\frac{V}{p}}}$ where U~N(0,1), v~$\chi_{(p)}$ (prove using Jacobian,T=U/(WP)$^{0.5}$, W=V
Uniform (a,b)	$\frac{1}{b-a}$	a≤x≤b	(b+a)/2	$(b-a)^2/12$	$\frac{e^{bt}-e^{at}}{(b-a)t}$	a=0,b=1 is case of the beta (1,1) X^2~Beta(1/2, 1) if X~Unif(0,1) Jth order statistic of U(0,1) ~Beta (j,n+j-1)

$f_Y(y)=f_X\left(g^{-1}(y)\right)\left|\frac{d}{dy}g^{-1}(y)\right| for\ y\in Y$ … $f(u_1,u_2)=f_{x_1,x_2}(h_1(u_1,u_2),h_2(u_1,u_2)|J|where\ J=\begin{vmatrix}\frac{dx_1}{du_1} & \frac{dx_1}{du_2}\\ \frac{dx_2}{du_1} & \frac{dx_2}{du_2}\end{vmatrix}$ $EX^n=d^n/dt^nM_x(t)|_{t=0}$, $\lim(1+a/n)^n=e^a$

Tranformations: i) Single RV, non-linear monotonic (x^2,logx) uses CDF technique ii) linear transformations, single/multiple RV's uses MGF iii) non-linear transformations, multiple RV's uses Jacobian

Cov(X,Y) = E[$x-\mu_x$,$Y-\mu_y$] = EXY-EXEY, Corr(X,Y) = Cov(X,Y)/($\sigma_x\sigma_y$), Var(X) = $EX^2-(EX)^2$, $EX=E_yE_x(X|Y)$); prove using joint expectation, VarX = E(Var(X|Y))+Var(E(X|Y)); prove adding/subtracting E(X|Y) to VarX=E($[X-EX]^2$)

Chebychev: $Pr(|x-\mu|<t\sigma^2)=1-1/t^2$, P(g(x)≥r)≤E(g(x))/r…Marginal Distributions:$p_x(x)=\sum_y p_{x,y}(x,y)=\int f_{x,y}(x,y)dy$, $s^2=\frac{1}{n-1}[\sum(x_i-\mu)^2-n(\bar{x}-\mu)^2]$, $(n-1)s^2/\sigma^2\sim\chi^2_{n-1}$, $T=U/\sqrt{V/p}$,V~ χ^2_p, U~N(0,1), U independent of V

Order Statistics: $x_{(n)}$ in Unif(0,θ), $f(y)=(ny^{n-1}/\theta^n)I_{(0,\theta)}(y)$ implies max as $n(F_x(y))^{n-1}f_x(y)$: $x_{(1)}$ in Unif(θ,1), $f(y)=(n(1-(y-\theta)/(1-\theta))^{n-1}(1/(1-\theta))I_{(\theta,1)}(y)$ implies min as $n(1-F_x(y))^{n-1}f_x(y)$: x_r: $f(y)=n!/((n-r)!(n-r-1)!)F_x(y)^{r-1}f_x(y)(1-F_x(y))^{n-r}$

$$\int|x-a|=-\int_{-\infty}^{a}(x-a)dx+\int_{a}^{\infty}(x-a)dx\quad\left(to\ minimize,take\ \frac{d}{da}\right)=0$$

Factorization Theorem: $f(\underline{X},\theta)=h_1(T(\underline{X}),\theta)h_2(\underline{X})$ implies $T(\underline{X})$ sufficient, Complete: $E_\theta[g(T(\underline{X})]=0$ for all θ implies $Pr[g(T(\underline{X}))=0]=1$,

Exponential Family: $f(\underline{X}|\theta)=h_1(\underline{X})h_2(\theta)\exp\{T_1(\underline{X})w_1(\theta)\ldots T_k(\underline{X})w_k(\theta)\}\ldots T_{1..k}(\underline{X})$ complete if $w_{1\ldots k}(\theta)$ form k-dimensional rectangle